

COMMON BUT UNAPPRECIATED SOURCES OF ERROR IN ONE, TWO, AND
MULTIPLE-COLOR PYROMETRY

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Introduction

Optical pyrometry is commonly used to measure temperatures above 1000 K in environments where thermocouples would be unstable or cause interference with the system under study. When great care is used, optical pyrometers can have an accuracy of ± 2 K or better. When used indiscriminately, errors in excess of ± 100 K are easily obtained. Although most of the sources of error have been known for some time, they are still often neglected. For the purposes of this paper the errors can be classified in one of three areas:

- i) Errors due to noise and non-linearity in the detection system or uncertainty about properties such as emissivity.
- ii) Errors due to stray radiation.
- iii) Errors due to the finite response speed of the detection system.

Background

All optical pyrometric systems are based on either Planck's law[1],

$$I_{\lambda} = \frac{2\epsilon_{\lambda}C_1}{\lambda^5(e^{C_2/\lambda T} - 1)}, \quad (1)$$

where I_{λ} is the intensity at wavelength λ , ϵ_{λ} is the emissivity of wavelength λ , T is the temperature, and C_1 and C_2 are the first and second radiation constants, or the approximation known as Wien's law[1],

$$I_{\lambda} = \frac{2\epsilon_{\lambda}C_1}{\lambda^5 e^{C_2/\lambda T}} \quad (2)$$

Most optical detectors generate a voltage proportional to the intensity. If one designates the voltage as V_λ , the temperature is calculated from V_λ as,

$$T = \frac{-C_2/\lambda}{\ln(V_\lambda) - \ln(G) - \ln(\epsilon_\lambda)} , \quad (3)$$

where the term G accounts for the detector gain, the optical bandwidth and other similar quantities.

The equivalent expression for two-color pyrometry is,

$$T = \frac{-C_2(1/\lambda_1 - 1/\lambda_2)}{\ln\left(\frac{V_{\lambda_1}}{V_{\lambda_2}}\right) - \ln\left(\frac{G_1}{G_2}\right) - \ln\left(\frac{\epsilon_{\lambda_1}}{\epsilon_{\lambda_2}}\right)} \quad (4)$$

In multiple-color pyrometry, a functional form of ϵ_λ vs. λ is assumed such as a linear fit, and the temperature is calculated from a least-squares fit of Planck's law to V_λ vs. λ data.

Noise Errors

If an error analysis is performed on Equation 3, one obtains[2],

$$\delta T = \frac{T^2}{C_2/\lambda} \left[\sum \left(\frac{\delta n}{n} \right)^2 \right]^{1/2} , \quad (5)$$

where δT is the error in T and the $\delta n/n$'s can represent the uncertainty in V_λ , ϵ_λ and any of the factors in G . The points to note are that for a constant uncertainty in the measurements, the error in the temperature increases as the square of the temperature and decreases linearly with λ . Figure 1 is a plot of the error in T as a function of T for a constant one-percent error in the measurement with λ as a parameter. The trends in T and λ stated above are clearly visible.

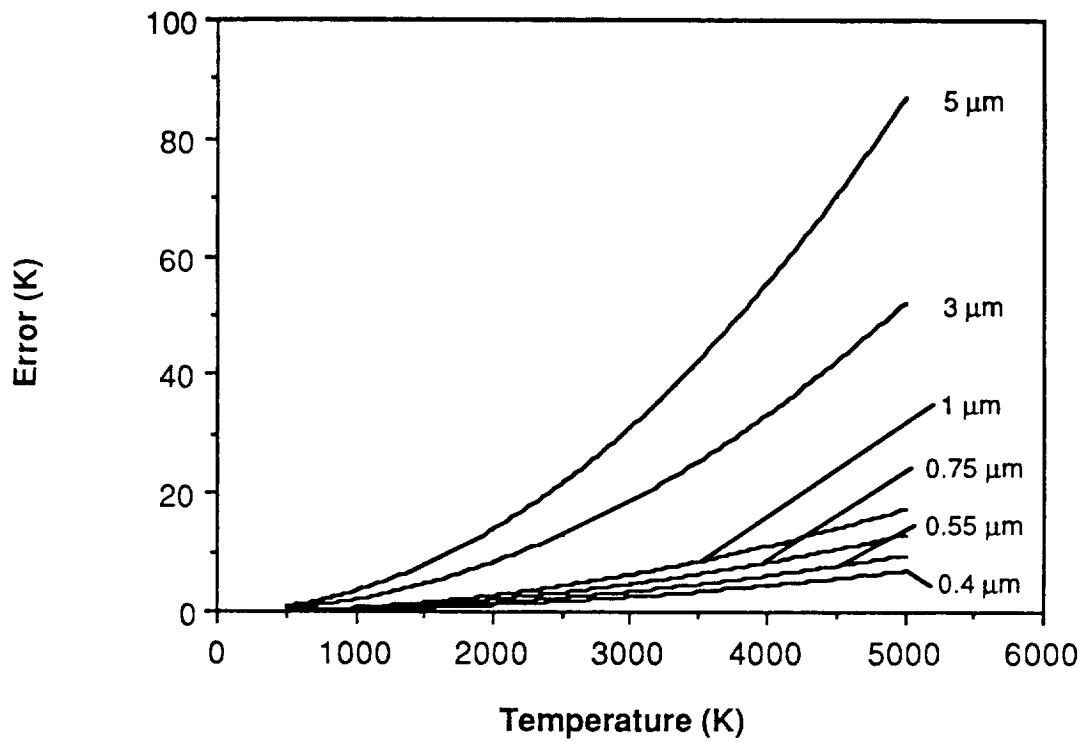


Figure 1 Calculated error in temperature for single color pyrometers for a 1% error in the signal

One effect that was neglected in Figure 1 is background noise or dark current. The background noise becomes a larger and larger fraction of the signal as the temperature drops. Figure 2 is equivalent to Figure 1 but includes this effect. Note the logarithmic scale on the Error axis. The 5 μ m, 3 μ m and 1 μ m curves were calculated using typical D^* values for indium-antimonide detectors. The 750 nm, 550 nm, and 400 nm curves were calculated using typical D^* values for photomultiplier tubes. The steep initial decrease in the error is due totally to the dark current. The gradual rise is the same as on Figure 1. The exact placement of the minimum depends on the actual D^* and on the field of view of the detector.

For a two-color pyrometer the error equation is[2],

$$\delta T = \frac{T^2}{C_2(1/\lambda_1 - 1/\lambda_2)} \left[\sum \left(\frac{\delta n}{n} \right)^2 \right]^{1/2} \quad (6)$$

where $\delta n/n$ is now the error in the ratio of ϵ_{λ_1} and ϵ_{λ_2} , G_1 to G_2 , *etc.*, as well as the noise inherent in V_{λ_1} and V_{λ_2} . The point to note here is that for a constant uncertainty in the measurement, a two-color pyrometer always has a greater error than the one color temperature calculated from the shorter wavelength signal, and the closer together the wavelengths are, the worse the error. The only reasons to use two-color pyrometers are when the target area is unknown or changing with time, the intensity at the two wavelengths is being attenuated equally by an intervening medium, or there is strong reason to believe that $\epsilon_{\lambda_1}/\epsilon_{\lambda_2}$ is more than $\frac{1/\lambda_1}{1/\lambda_1 - 1/\lambda_2}$ times as accurate as ϵ_{λ_1} , where λ_1 is the shortest

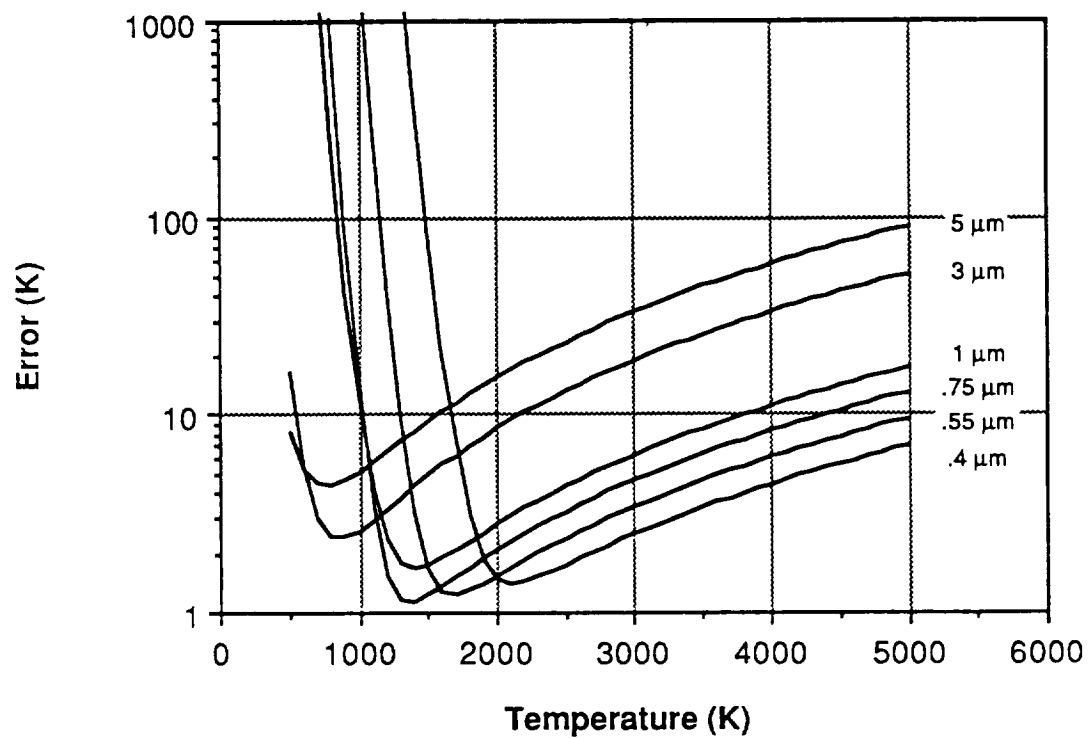


Figure 2 Calculated error in temperature for single color pyrometers for a 1% error in the signal and accounting for dark current or zero-signal noise

wavelength. Otherwise single color pyrometry is superior. Figure 3 is a plot of δT vs. T for some two-color pairs for a constant one-percent error. The 581 nm - 545 nm pair was used in reference [3] to measure the temperature of burning coal particles. The background noise in a two-color system will usually be dominated by the behavior of the shortest-wavelength detector.

The error analysis of multiple-color pyrometry is beyond the scope of this paper but some facts can be established. Even with good data the minimum in the least-squares error is very broad. The chosen functional form of ϵ_λ can have strong correlation effects with the Planck function. In computer simulations the uncertainty is typically one-to-five percent in the temperature[4] On actual data where the emissivity is not known the uncertainties are typically ten-to-twenty percent or greater[5].

Stray Radiation Errors

An object that is not perfectly emissive reflects or scatters a portion of the radiation incident on it. If this reflected or scattered radiation reaches the pyrometric detectors, they will indicate an incorrect temperature[6]. If the direct radiation from the target is i , and the stray background reaching the detector is i_b , the measured temperature for a one-color pyrometer is,

$$T_m = \frac{-C_2/\lambda}{\ln(1 + i_b) - \ln(\epsilon_\lambda)} \quad (7)$$

In general the calculation of i_b requires a knowledge of the angular emissivity of the interfering background, the angular reflectivity of the target, and the viewfactors in the system. For

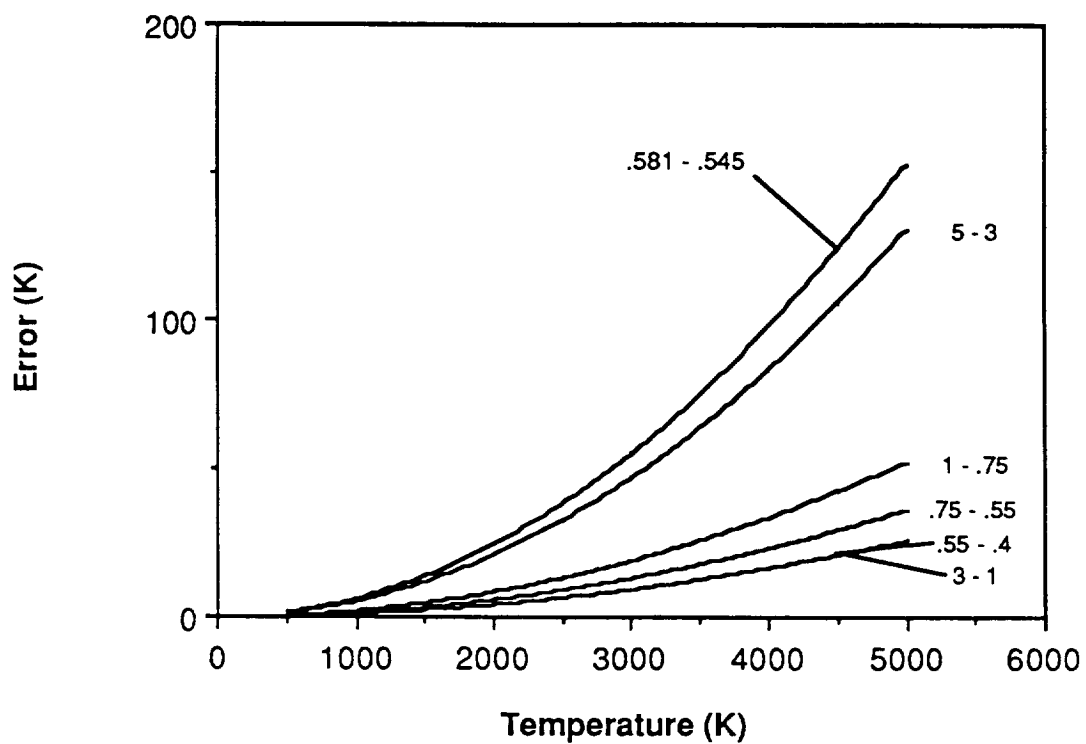


Figure 3 Calculated error in temperature for two color pyrometers for a 1% error in the signal

the much simplified case where the background and the target are diffuse reflectors and emitters and the background surrounds the target, the measured temperature can be calculated as[6]

$$T_m = \frac{-C_2/\lambda}{\ln \left[e^{-C_2/\lambda T} + \frac{(1-\epsilon)\epsilon_b}{\epsilon} e^{-C_2/\lambda T_b} \right]}, \quad (8)$$

where T and ϵ are the temperature and emissivity of the target and T_b and ϵ_b are the temperature and emissivity of the background.

Figures 4, 5 and 6 are plots of the measured temperature as a function of the background temperature as calculated by Equation 8 with the target temperature as a parameter. When the target is a good emitter and the background is a weak emitter, as in Figure 4, the background doesn't affect the measured temperature until the background temperature is substantially above that of the target. When the target and background emissivities are equal to 0.5, as in Figure 5, the background has some effect even at temperatures below that of the target. The effect of the background becomes more pronounced at higher temperatures as predicted by Equation 5. When the background is much more emissive than the target, as in Figure 6, the background must be much cooler than the target or the measured temperature will be grossly in error.

The equivalent general expression for two colors is,

$$T_m = \frac{-C_2(1/\lambda_1 - 1/\lambda_2)}{\ln \left(\frac{\epsilon_{\lambda_1} + \epsilon_{b\lambda_1}}{\epsilon_{\lambda_2} + \epsilon_{b\lambda_2}} \right) - \ln \left(\frac{\epsilon_{\lambda_1}}{\epsilon_{\lambda_2}} \right)}, \quad (9)$$

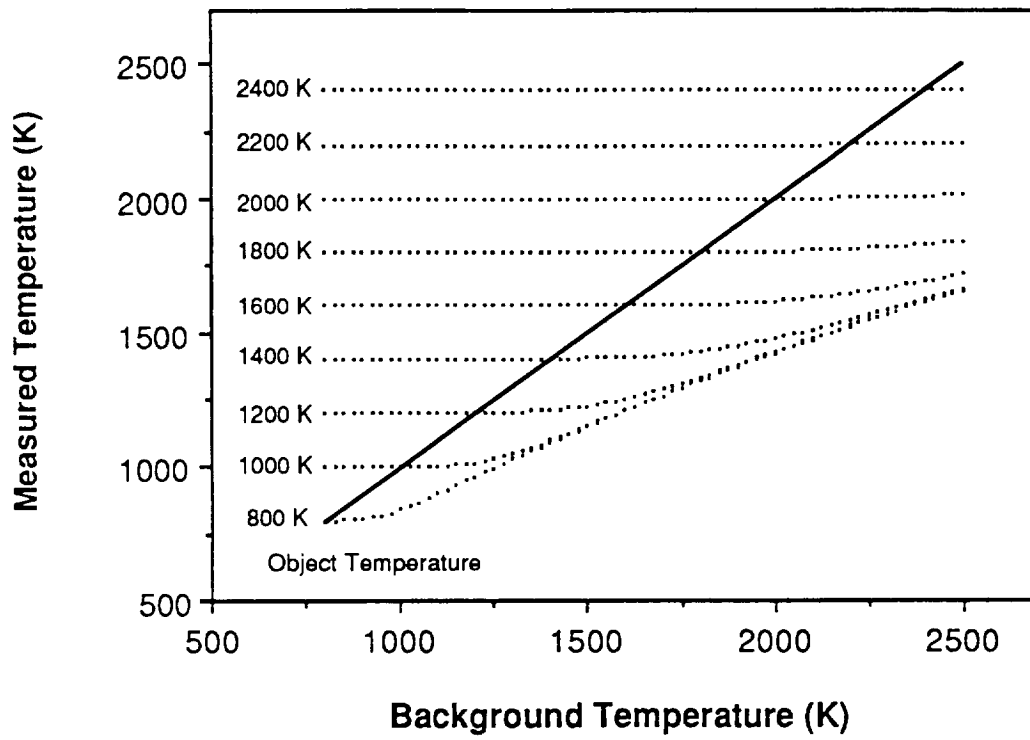


Figure 4 The temperature measured by a 650 nm single-color pyrometer for a given object temperature as a function of the background temperature, $\epsilon = 0.9$, $e_b = 0.1$

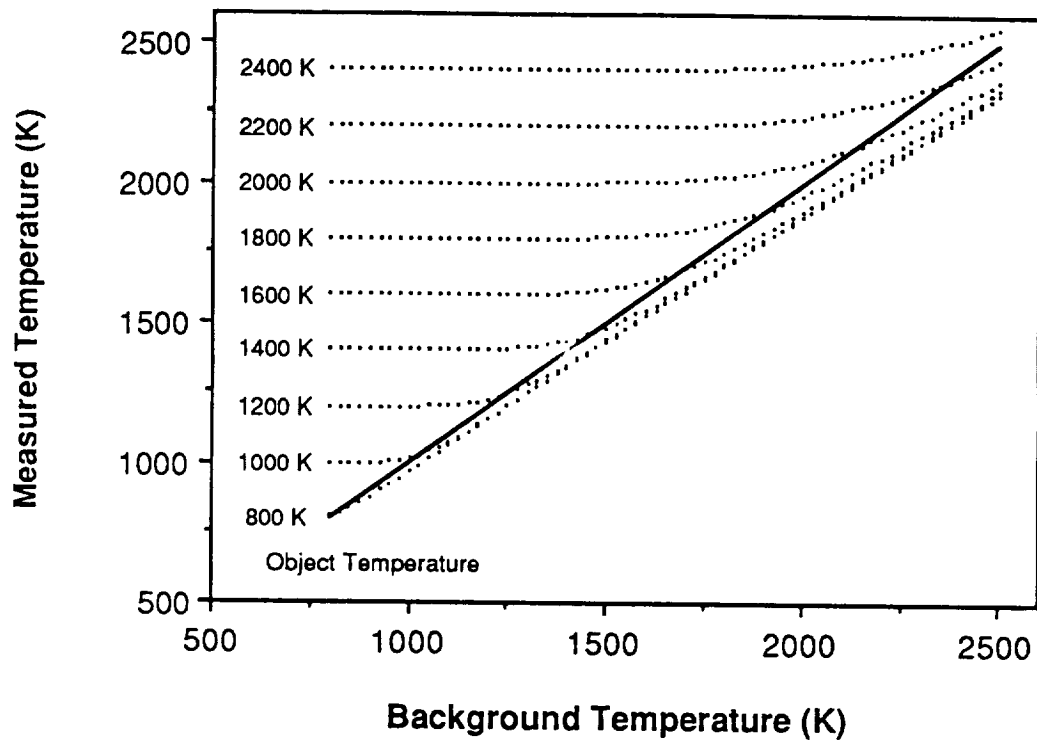


Figure 5 The temperature measured by a 650 nm single-color pyrometer for a given object temperature as a function of the background temperature, $\epsilon = 0.5$, $e_b = 0.5$

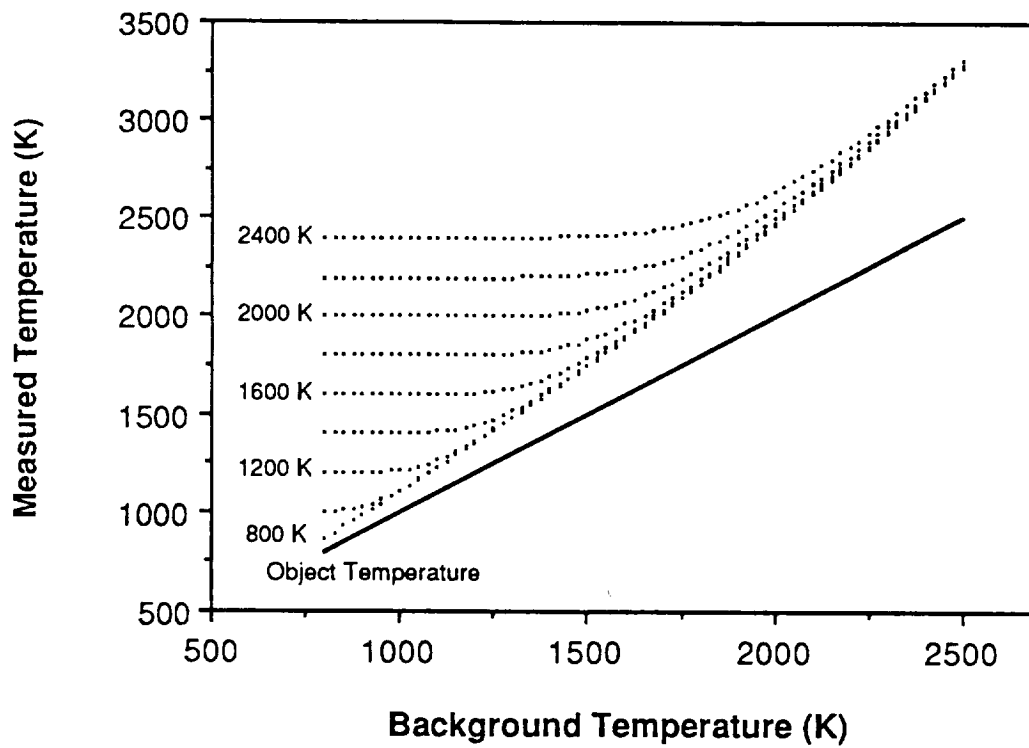


Figure 6 The temperature measured by a 650 nm single-color pyrometer for a given object temperature as a function of the background temperature, $\epsilon = 0.1$, $e_b = 0.9$

and the general case involves a knowledge of the angular-spectral emissivities and reflectivities at λ_1 and λ_2 , and all of the relevant viewfactors. For the much-simplified case of background and target being diffuse gray-bodies and the background surrounding the target the measured temperature can be calculated as,

$$T_m = \frac{-C_2(1/\lambda_1 - 1/\lambda_2)}{\ln \left[\frac{e^{-C_2/\lambda_1 T} + \frac{(1-\epsilon)\epsilon_b}{\epsilon} e^{-C_2/\lambda_1 T_b}}{e^{-C_2/\lambda_2 T} + \frac{(1-\epsilon)\epsilon_b}{\epsilon} e^{-C_2/\lambda_2 T_b}} \right]} \quad (10)$$

where λ_1 and λ_2 are the two pyrometric wavelengths.

Figures 7, 8 and 9 are plots of the measured temperature as a function of the background temperature as calculated by Equation 10 with the target temperature as a parameter. Because of the gray-body assumption, the measured temperature always asymptotically approaches the background temperature at high T_b . In Figure 7 the target is much more emissive than the background. Consequently, the background doesn't really affect the measurements until the background temperature is substantially above the target temperature. In Figure 8 the emissivities are both equal to 0.5. The measured temperature is affected much sooner by the background and at higher target temperatures a slight dip is found in T_m . In Figure 9 the background is much more emissive than the target and the dip in T_m is much more pronounced. The effect is counterintuitive and is due to the nonlinear nature of the Planck function.

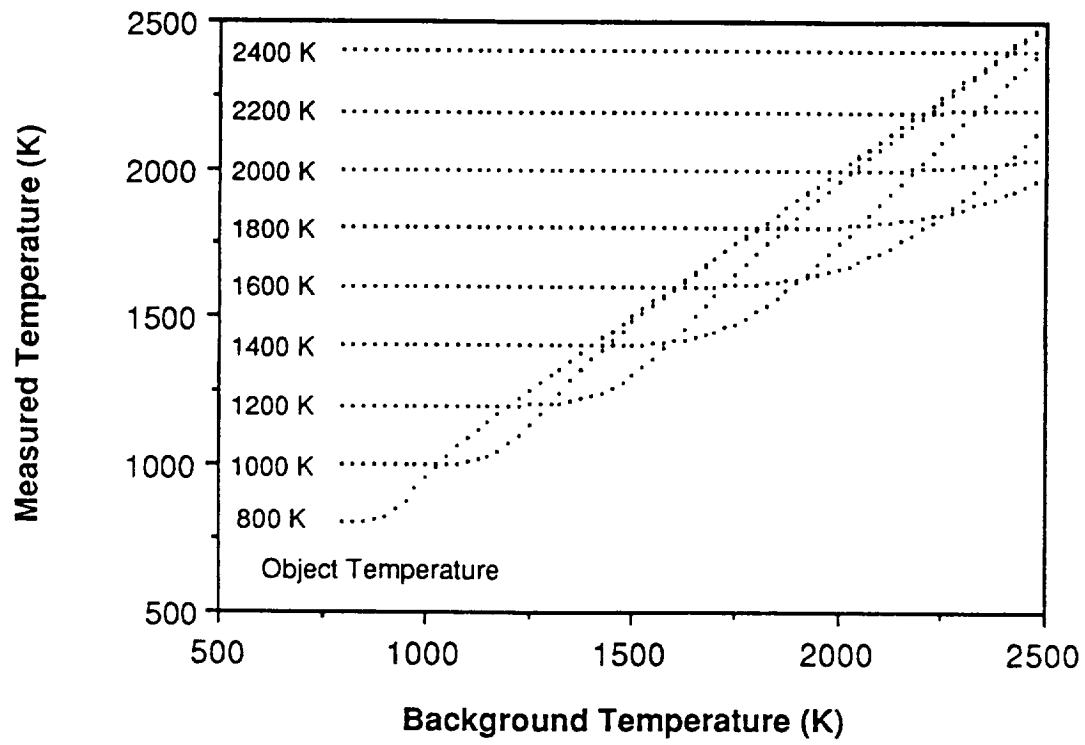


Figure 7 The temperature measured by a 650 nm - 550 nm two-color pyrometer for a given object temperature as a function of the background temperature, $\epsilon = 0.9$, $e_b = 0.1$

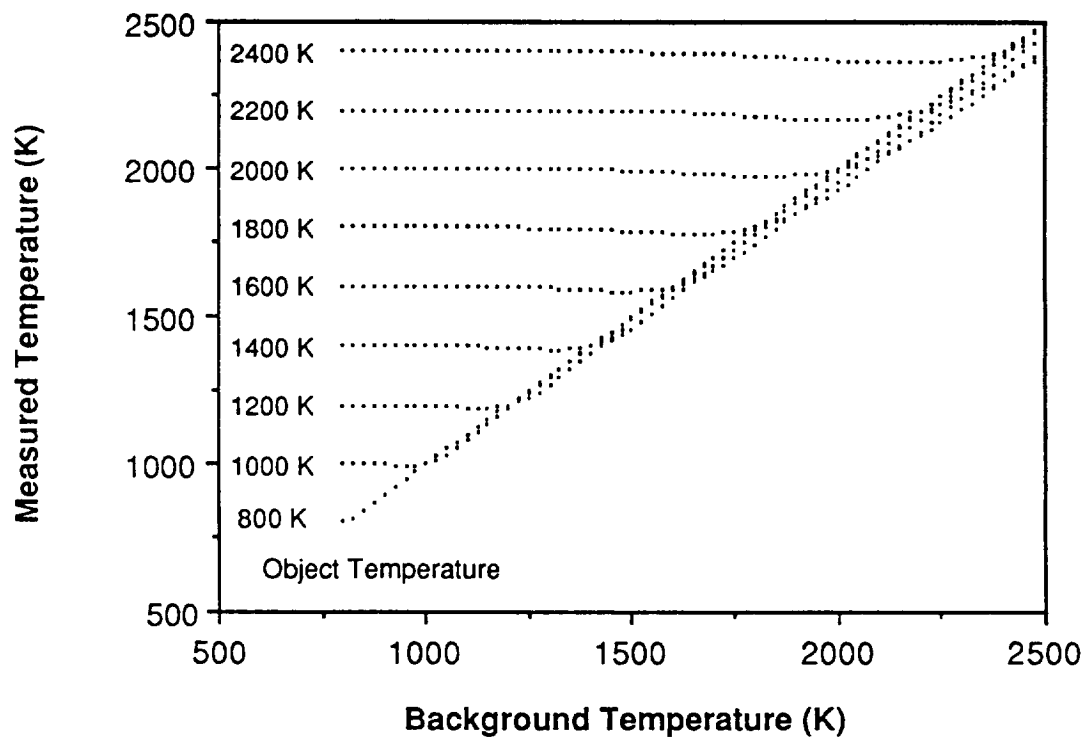


Figure 8 The temperature measured by a 650 nm - 550 nm two-color pyrometer for a given object temperature as a function of the background temperature, $\varepsilon = 0.5$, $e_b = 0.5$

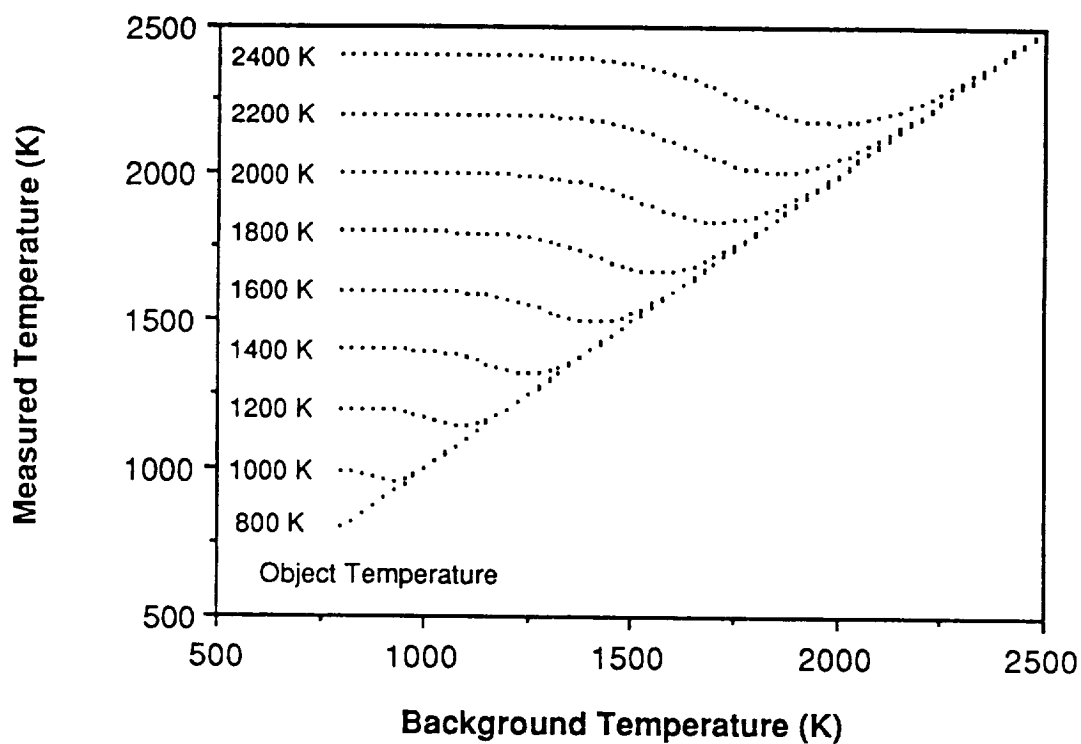


Figure 9 The temperature measured by a 650 nm - 550 nm two-color pyrometer for a given object temperature as a function of the background temperature, $\varepsilon = 0.1$, $e_b = 0.9$

The calculations for the general case are much more involved, especially if curved specular surfaces are present, but the trends are generally the same. One would prefer a hot emissive object in a very cold background.

The background radiation in a multi-wavelength system can masquerade as a wavelength-dependent emissivity and the magnitude and sign of the resultant error must be evaluated on a case-by-case basis and is not subject to heuristic interpretation.

Speed of Response Errors

In most pyrometric systems, the detector response time is considered fast with respect to the measured system's response time. The question of how fast is fast enough does not have a simple answer. An analysis performed in reference [7] of the transient response of a temperature measurement system shows that in some cases the transient response is enhanced in optical pyrometers and in some cases is retarded.

Most detector-system responses can be modeled as a first-order system, which is characterized by a unit step response of $1 - e^{-t/\tau}$, where t is the time and τ is the characteristic response time of a first-order system. For linear systems with higher order responses, the general trends shown below hold but the specific shape of the response is different. In a first-order system, the voltage response of a single detector for a step change in intensity from i_1 to i_2 (corresponding to temperatures T_1 and T_2) is given by

$$\frac{V(t)}{K} = i_2 + (i_1 - i_2)e^{-t/\tau} \quad (11)$$

where Wien's law is assumed and K is a constant that depends on the various gain factors. The temperature calculated from $V(t)$ has the transient response

$$T(t) = \frac{-C_2/\lambda}{\ln[e^{-C_2/\lambda T_2} + (e^{-C_2/\lambda T_1} - e^{-C_2/\lambda T_2})e^{-t/\tau}]} \quad (12)$$

For rises in temperature, the pyrometer temperature responds more quickly than expected, with the greatest enhancements for the largest ΔT 's. For temperature drops, the response is retarded, with the effect being greatest for the largest ΔT 's. These effects are due to the strong nonlinearity of i with respect to T .

The effect with two-color pyrometry shows the same general behavior but is even more dramatic. The equation for temperature analogous to Equation 12 is

$$T(t) = \frac{-C_2(1/\lambda_1 - 1/\lambda_2)}{\ln \left[\frac{e^{-C_2/\lambda_a T_2} + (e^{-C_2/\lambda_a T_1} - e^{-C_2/\lambda_a T_2})e^{-t/\tau}}{e^{-C_2/\lambda_b T_2} + (e^{-C_2/\lambda_b T_1} - e^{-C_2/\lambda_b T_2})e^{-t/\tau}} \right]} \quad (13)$$

An examination of the limiting cases can help to understand the much more pronounced effect with two colors than with one color. If perfect noiseless detectors are assumed and the temperature is permitted to step from an initial temperature of $T_1 = 0$ K up to T_2 , the single color temperature from Equation 12 would have the response

$$T(t) = \frac{-C_2/\lambda}{\ln[e^{-C_2/\lambda T_2}(1 - e^{-t/\tau})]} \quad (14)$$

whereas the two-color response from Equation 13 would be

$$T(t) = \frac{-C_2(1/\lambda_1 - 1/\lambda_2)}{\ln \left[\frac{e^{-C_2/\lambda_a T_2}(1 - e^{-t/\tau})}{e^{-C_2/\lambda_b T_2}(1 - e^{-t/\tau})} \right]} = T_2 \quad (15)$$

In other words, the ratio of the two signals responds instantly even though the signals themselves take much longer to respond. As T_1/T_2 approaches zero, the system response more closely approaches a perfect step change.

If, however, the temperature is permitted to drop from an initial temperature of T_1 to $T_2 = 0$ K, the single-color response is

$$T(t) = \frac{-C_2/\lambda}{\ln [e^{-C_2/\lambda T_1} e^{-t/\tau}]} \quad (16)$$

whereas the two-color response is

$$T(t) = \frac{-C_2(1/\lambda_1 - 1/\lambda_2)}{\ln \left[\frac{e^{-C_2/\lambda_a T_1} e^{-t/\tau}}{e^{-C_2/\lambda_b T_1} e^{-t/\tau}} \right]} = T_1 \quad (17)$$

The two-color system would indicate the initial temperature forever. The above situation corresponds to T_1/T_2 approaching infinity.

The calculated effect on a typical pyrometry system is seen in Figure 10. In this simulated example a two-color optical pyrometer with center wavelengths of 750 nm and 450 nm and a linear response time of 10 ms is used to measure the temperature of an object initially at 1000 K. The temperature of the object is instantaneously raised to 1500 K and maintained for 50 ms. The temperature is then instantaneously reduced to 1000 K and held there. On the figure the $\exp(-\text{time}/\tau)$ curve corresponds to the temperature a thermocouple with a response time of 10 ms would

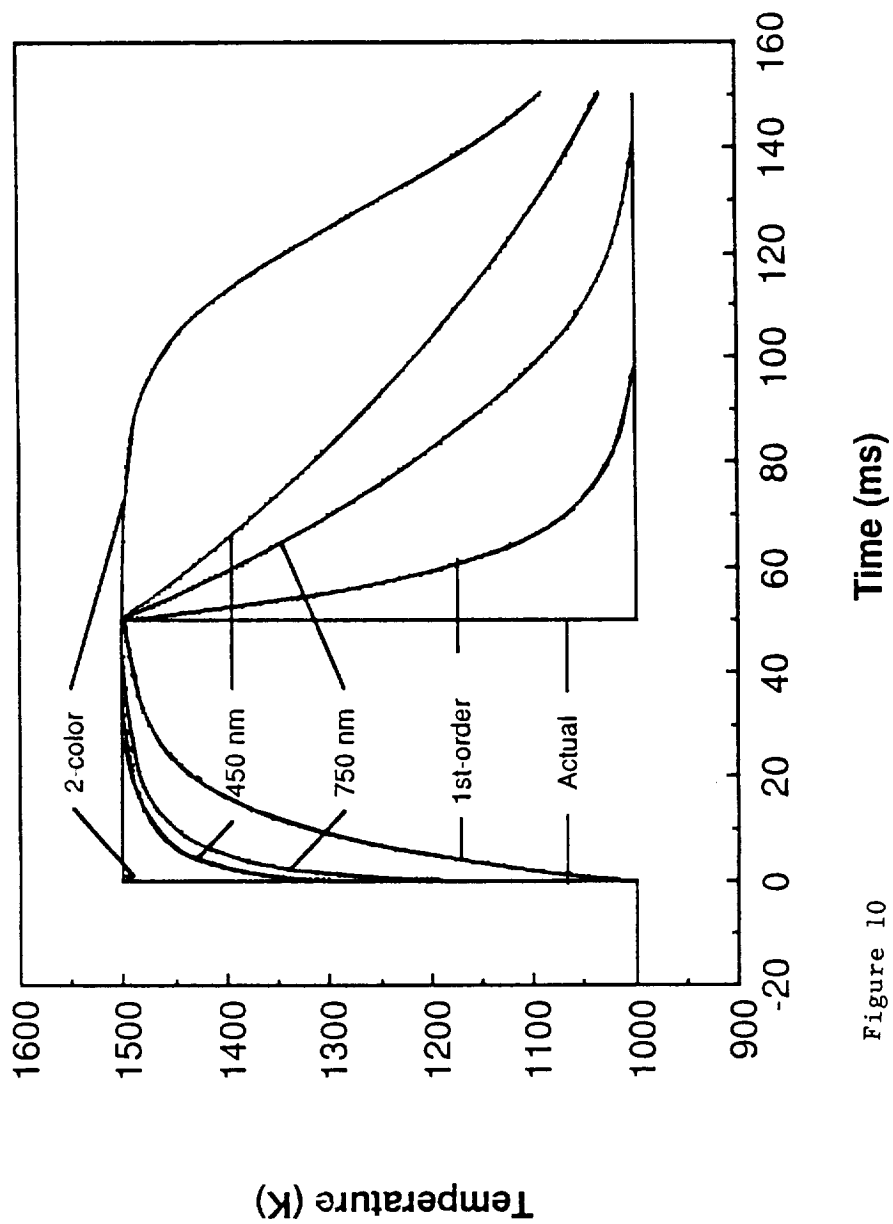


Figure 10

Calculated response of a linear thermo-couple, a 750 nm single-color pyrometer, a 450 nm single-color pyrometer, and a 750 nm - 450 nm two-color pyrometer, all with a first-order response time of 10 ms to a 50 ms temperature pulse.

indicate. The next two curves in ascending order are for the single-color temperatures at 750 nm and 450 nm. The top curve is the two-color temperature.

For the temperature rise the single-color temperatures respond more quickly than the linear thermocouple. The response is faster for the shorter wavelengths. The two-color temperature tracks the step so closely that it is difficult to distinguish from the true temperature. Clearly the two-color response is excellent.

The four curves maintain their relative orderings with respect to temperature for the temperature drop. The two-color temperature response, which was previously excellent, is now almost unacceptably slow. One must be careful when measuring temperature decreases with an optical pyrometer.

If signal-processing is applied to the signals, an option for improving the behavior is available. If one can take the logarithm of the signal before the filtering or averaging is performed, the response becomes symmetric with respect to the reciprocal of the temperature. In other words, if the first-order system processes $\log(i)$ instead of i , the transient response for both one- and two-color systems is

$$\frac{1}{T(t)} = \frac{1}{T_2} + \left(\frac{1}{T_1} - \frac{1}{T_2} \right) e^{-t/\tau} \quad (18)$$

Actual results from the measurement of the transient temperature of a 60 μm platinum thermocouple heated by a laser are presented in Figures 11 and 12. In Figure 11 the thermocouple was initially at 1173 K. At time $t = 0$ the temperature was raised

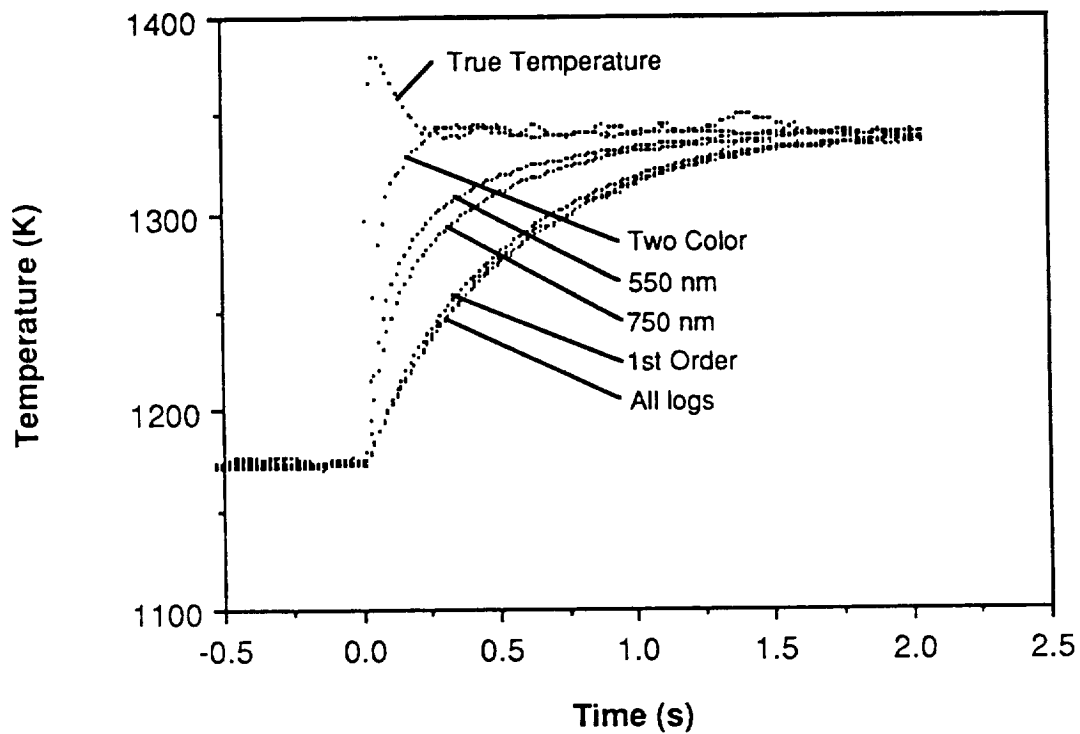


Figure 11 Experimental transient temperature behavior for a linear temperature measurement system, a two-color pyrometer, two one-color pyrometers, and all three pyrometers using the log of the intensity, for a temperature jump

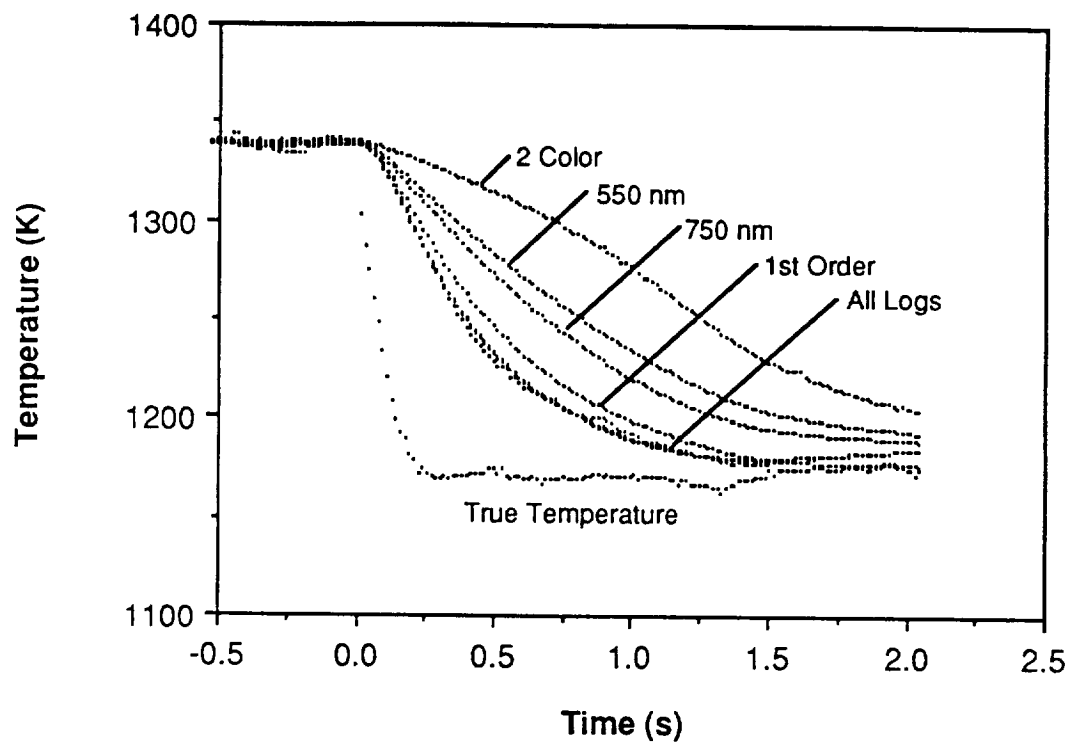


Figure 12 Experimental transient temperature behavior for a linear temperature measurement system, a two-color pyrometer, two one-color pyrometers, and all three pyrometers using the log of the intensity, for a temperature drop

to 1341 K. The signal from the thermocouple and the signals from the photomultiplier tubes were recorded.

The actual temperature was calculated from the raw thermocouple data without any filtering. The data from the thermocouple and the two optical channels were filtered digitally with the exact equivalent of a first-order analog system to demonstrate the effect of a first-order response. The system time constant was set at $\tau = 0.509$ s. In Figure 11, as expected, both single-color temperatures responded more quickly than the filtered thermocouple response and are properly ordered with respect to wavelength. The two-color response was very close to the true response. It also shows more noise than the single-color temperatures, as one would expect from the error analysis mentioned above[2,8,9]. The logarithms of the signals were processed as well. All three temperatures calculated from the log-signals fell on the same curve and were very close to the linear first-order response.

A representative set of results for a temperature drop are plotted in Figure 12. In this experiment the thermocouple was initially at 1338 K and at time $t = 0$ the temperature was lowered to 1172 K. As expected, the log-intensity temperatures have the fastest response and the the filtered thermocouple is just slightly slower, the 750 nm temperature and the 550 nm are intermediate and the two-color response is the slowest. The theoretical responses predicted by Equations 12, 13 and 18 are not shown but agree very well with the experimental points.

Further insight can be gained from a frequency analysis of the two-color response. Because the system is nonlinear, the frequency response cannot be uniquely defined but is different for each combination of wavelengths and magnitude of temperature-oscillation. The results for a two-color pyrometer with wavelengths 850 nm and 550 nm at one frequency are given in Figure 13. Here a sinusoidally varying temperature is measured by a thermocouple and by a two-color pyrometer, each with response-time τ . The true temperature is given by $T = 1000\text{ K} + 100\text{ K} \sin(\omega t)$ where $\omega\tau = 1$. The two-color signal has an average temperature that is 50 K above the mean of the true temperature and has an amplitude roughly half that of the thermocouple temperature.

Figure 14(a) is a plot of the difference between the pyrometer average temperature and the true mean temperature as a function of frequency. Figure 14(b) is a Bode-type plot of the root-mean-square (rms) amplitude of the time-varying component of the pyrometer temperature about the average pyrometer temperature as a function of frequency. The rms response of a first-order linear system is shown for comparison.

The rms signal of the pyrometer decreases more rapidly with increasing frequency than for a first-order system, and it also displays an offset which the first-order system does not display. When measuring rapidly fluctuating temperatures, an optical pyrometer will not show the correct average temperature, and will under-indicate the magnitude of the fluctuations by a larger amount than a linear system with the same response time would do.

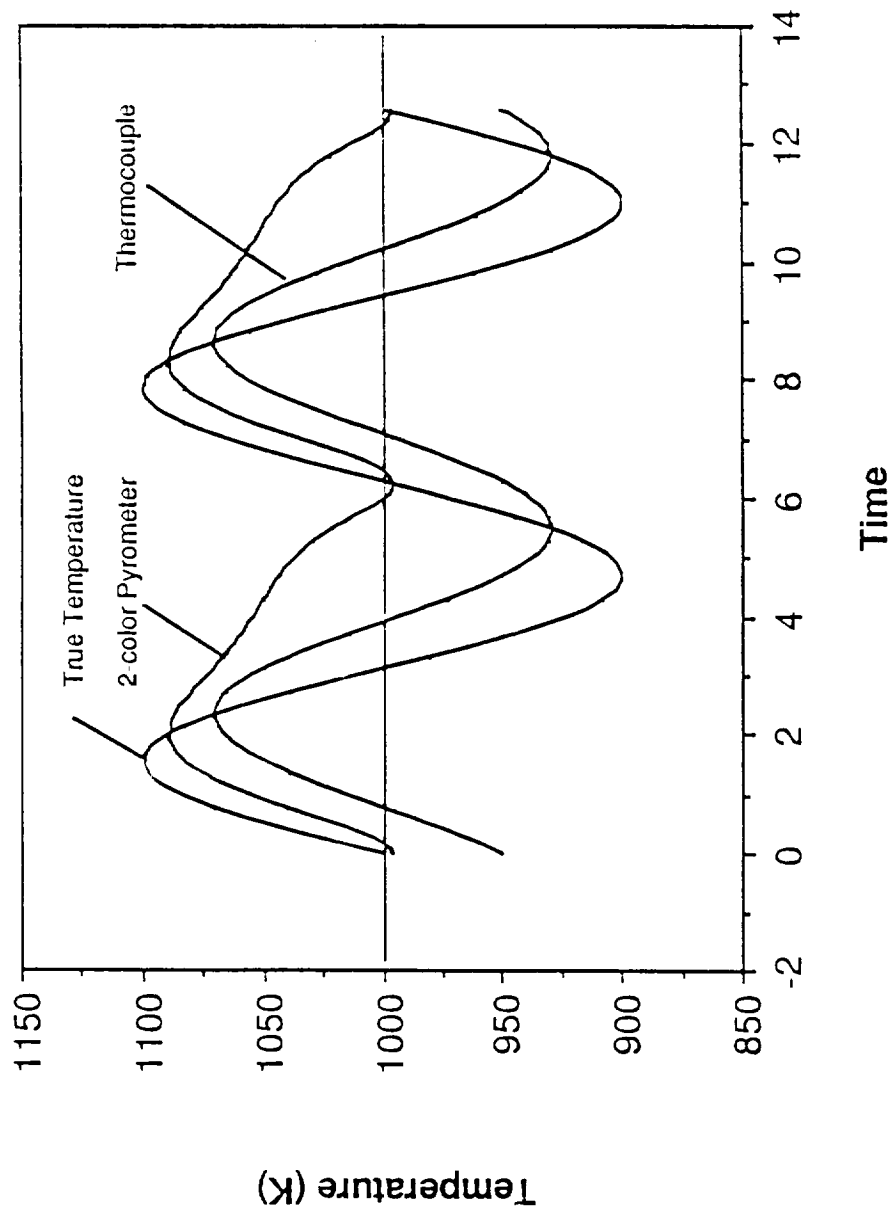


Figure 13 The actual temperature and the temperature reported by a linear device and a two-color pyrometer for an object with a sinusoidally varying temperature

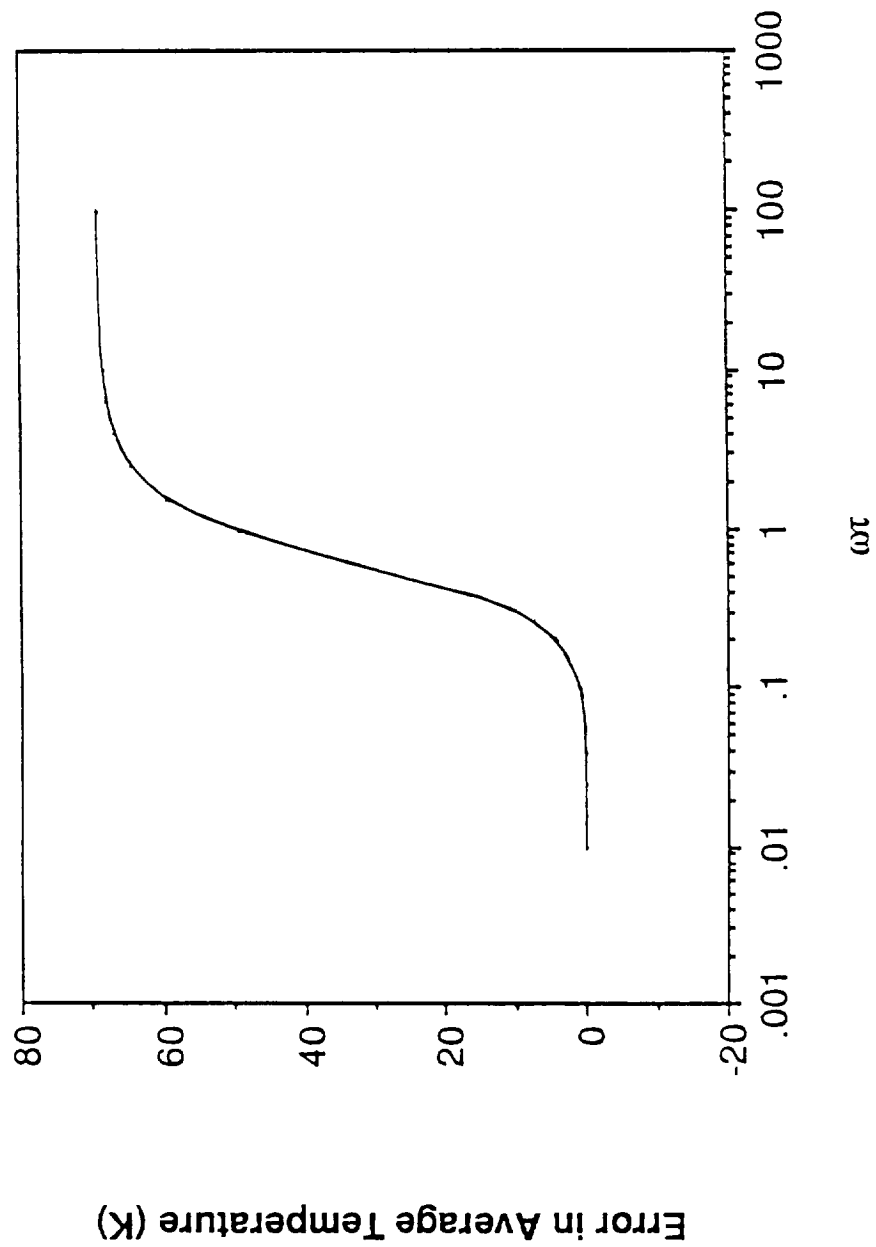


Figure 14(a) The error in the average temperature indicated by a two-color pyrometer for an object with a sinusoidally varying temperature as a function of the frequency of the sinusoid

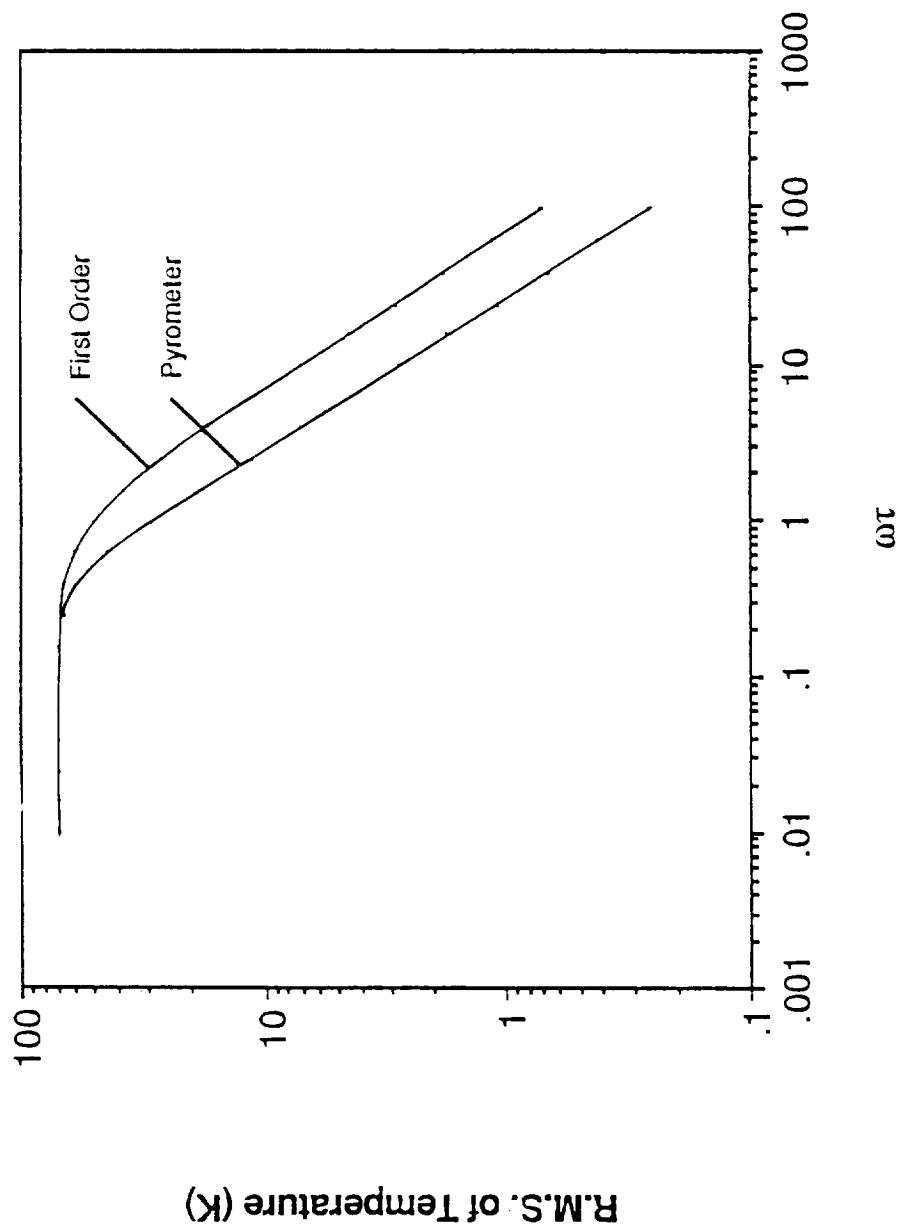


Figure 14(b) A comparison of the root-mean-square amplitude of the measured temperature for a linear system and a two-color pyrometer for an object with a sinusoidally varying temperature as a function of the frequency of the sinusoid. The amplitude of the actual temperature does not change with frequency.

The effect of different τ 's for the two detectors has also been investigated, and it was found that differences of up to three percent produce relatively minor perturbations and can be neglected.

The response time of a two-color system must be at least 100 times faster than the expected response time of the object being measured to ensure that reasonably accurate measurements are being made. Rapid temperature increases, which are often the features of interest, are measured quite readily by a slow two-color pyrometer. But measurement of temperature fluctuations will show gross errors if the system response is not fast enough. Because the trends are the result of nonlinearities between i and T and of the fact that the ratio of two signals responds much differently than do its constituent signals, these trends hold for systems with other than ideal first-order responses. They are also independent of whether the temperature calculation is performed by analog circuitry or digitally in a computer. For signal-processing applications, the log of the intensity and not the raw intensity data should be used.

Summary

The most common sources of error in optical pyrometry have been examined. They can be classified as either noise and uncertainty errors, stray radiation errors, or speed-of-response errors. Through judicious choice of detectors and optical wavelengths the effect of noise errors can be minimized, but one should strive to determine as many of the system properties as possible. Careful

consideration of the optical-collection system can minimize stray-radiation errors. Careful consideration must also be given to the slowest elements in a pyrometer when measuring rapid phenomena.

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